3. Conic Section

Learning Objectives

After successful completion of this chapter, the reader should be able to learn and appreciate:

- Introduction to the conic sections
- Types of conic sections

3.0 Introduction

Hypatia
(370 – 415 AD)

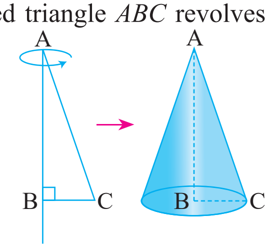
The Greeks discovered conic sections sometime between 600 and 300 BC. By the beginning of the Alexandrian period, enough was known about conic for Apollonius (262-190 BC) to produce an eight-volume work on the subject. Later toward the end of Alexandrian period, Hypatia wrote a textbook entitled "On the Conics of Apollonius". Her death marked the end of major mathematical discoveries in Europe for several hundred years.

Hypatia was probably the first woman to have profound impact on the survival of early thought in mathematics.

3.1 Cone

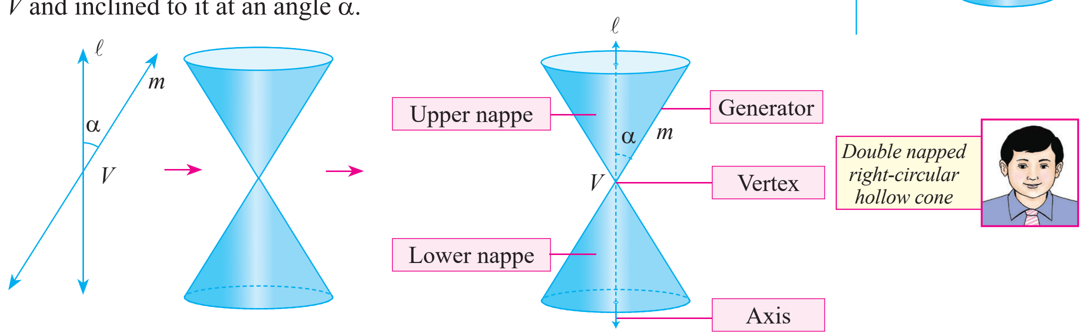
In the adjoining figure, ΔABC is a right angled triangle. If the right angled triangle ABC revolves about the side $AB = h$, which is kept fixed, then it generates a cone.

Thus, rotating a right angled triangle around one of its shorter sides (making the side as an axis) will produce a right circular cone.



3.2 Conic Section

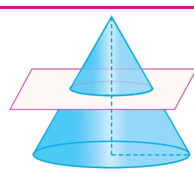
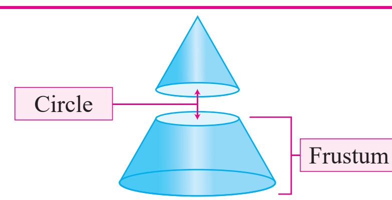
Let ℓ be a fixed vertical line and m be another line intersecting it at a point V and inclined to it at an angle α.



Suppose, we rotate the line m around the line ℓ in such a way that the angle α remains constant. Then the surface generated is a double-napped right circular hollow cone.

The point V is called the vertex and the line ℓ is the axis of the cone. The rotating line m is called a generator of the cone. The vertex separates the cone into two parts called nappes.

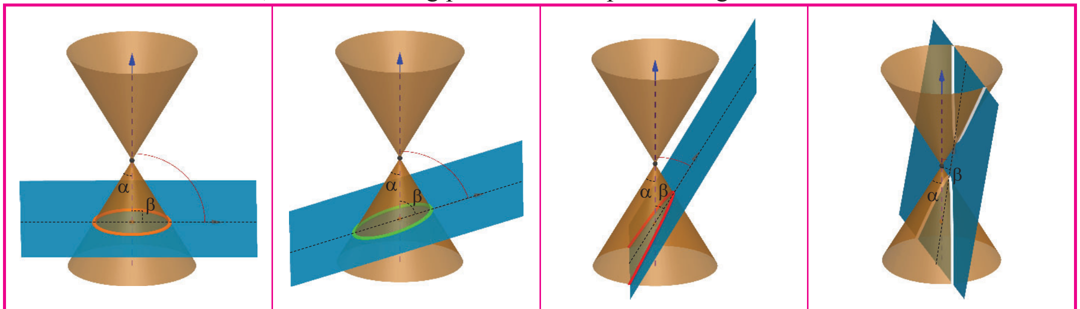
Look at the following figures.

Plane is parallel to the base		

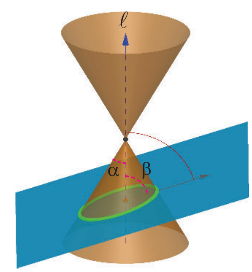
Here, circle is produced by the intersection of a plane and conical surface. If we take the intersection of a plane with a cone, the section so obtained is called a conic section. Thus, *conic sections are the curves obtained by intersecting a right circular cone by a plane.*

3.3 Circle, ellipse, parabola and hyperbola

Each conic section can be described as the intersection of a plane and a double-napped cone. Following are the four basic conics, the intersecting plane does not pass through the vertex of the cone.



Circle ($\beta = 90°$) | Ellipse ($\alpha < \beta < 90°$) | Parabola ($\beta = \alpha$) | Hyperbola ($0 \le \beta < \alpha$)



We obtain different kinds of conic sections depending on the position of the intersecting plane with respect to the cone and by the angle made by it with the vertical axis of the cone. Let β be the angle made by the intersecting plane with the vertical axis of the cone.

The intersection of the plane with the cone can take place either at the vertex of cone or at any other part of the nappe either below or above the vertex.

When the plane cuts the nappe (other than the vertex) of the cone, we have the following situations:

(a) When $\beta = 90°$ then the section is a circle.

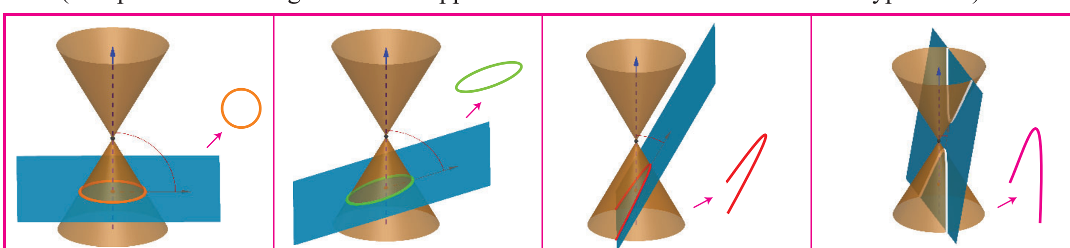
(b) When $\alpha < \beta < 90°$ then the section is an ellipse.

(c) When $\beta = \alpha$ then the section is a parabola.

(In each of the above three situations, the plane cuts entirely across one nappe of the cone.)

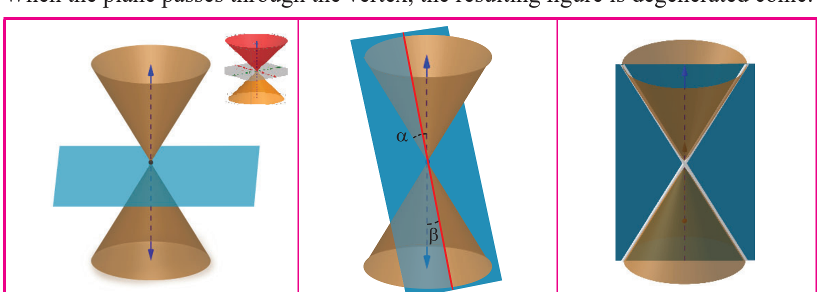
(d) When $0 < \beta < \alpha$ then the section is a hyperbola.

(The plane cuts through both the nappes and the curves of intersection is a hyperbola.)



(a) | (b) | (c) | (d)

When the plane passes through the vertex, the resulting figure is degenerated conic.



Point degenerate conics | Line | Two intersecting lines